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LETTER TO THE EDITOR

Critical behaviour of 3D systems with long-range correlated quenched defects

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Abstract. A field-theoretic description of the critical behaviour of systems with quenched defects obeying power law correlations $\sim |\boldsymbol{x}|^{-a}$ for large separations $\boldsymbol{x}$ is given. Directly, for three-dimensional systems and for different values of the correlation parameter, $2 \leqslant a \leqslant 3$, a renormalization analysis of the scaling function in the two-loop approximation is carried out, and the fixed points corresponding to the stability of various types of critical behaviour are identified. The obtained results essentially differ from results evaluated by a double ε, δ-expansion. The critical exponents in the two-loop approximation are calculated with the use of the Padé–Borel summation technique.

In recent years, much effort has been devoted to investigating the critical behaviour of solids containing quenched defects. In most papers considerations have been restricted to the case of point defects with small concentrations so that the defects and corresponding random fields have been assumed to be Gaussian distributed and δ-correlated.

For the first time in the work of Weinrib and Halperin (WH) [1] they have been offered a model of the critical behaviour of a disordered system in which the correlation function of the random local transition temperature $g(\boldsymbol{x}-\boldsymbol{y}) = \langle\langle T_c(\boldsymbol{x})T_c(\boldsymbol{y})\rangle\rangle - \langle\langle T_c(\boldsymbol{x})\rangle\rangle^2$ falls off with distance as a power law $\sim |\boldsymbol{x}-\boldsymbol{y}|^{-a}$. They showed that for $a \geqslant d$ long-range correlations are irrelevant and the usual short-range Harris criterion [2] $2 - d\nu_o = \alpha_o > 0$ of influence of δ-correlated point defects is realized, where d is the spatial dimension, and ν_o and α_o are the correlation-length and the specific-heat exponents of the pure system. For $a < d$ the extended criterion $2 - a\nu_o > 0$ of the influence of disorder on the critical behaviour was established. As a result, a wider class of disordered systems, not only the three-dimensional (3D) Ising model with δ-correlated point defects, can be characterized by a new type of critical behaviour. So, for $a < d$ a new long-range (LR) disorder stable fixed point (FP) of the renormalization group recursion relations for systems with a number of components of the order parameter $m \geqslant 2$ was discovered. The critical exponents were calculated in the one-loop approximation using a double expansion in $\varepsilon = 4 - d \ll 1$ and $\delta = 4 - a \ll 1$. In the case $m = 1$ the accidental degeneracy of the recursion relations in the one-loop approximation did not permit them to find LR disorder stable FP, but a change in critical behaviour of the model from the short-range (SR) to the LR-correlation type was predicted for $\delta > \delta_c = 2(6\varepsilon/53)^{1/2}$. Korzhenevskii *et al* [3] proved the existence of the LR disorder stable FP for the one-component WH model and also found characteristics of this type of critical behaviour. Also, they considered a very interesting

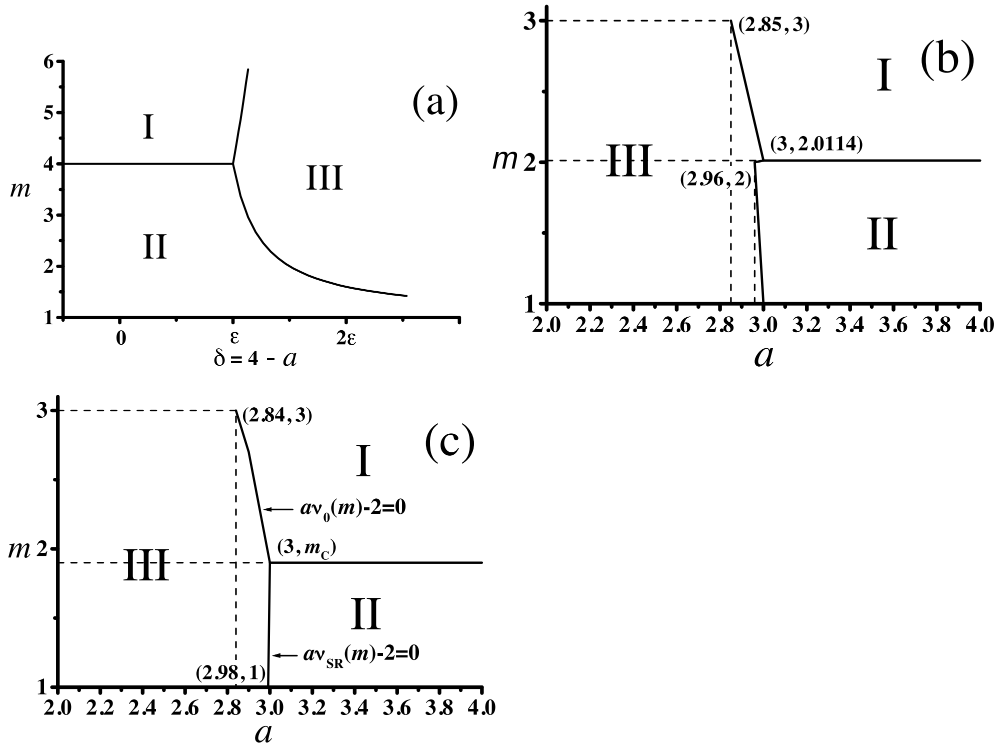

Figure 1. Regions of the various types of critical behaviour, which have been determined: (*a*) in [1] on the basis of the double ε, δ-expansion; (*b*) in the present paper with use of the field-theoretic description in a two-loop approximation for the 3D WH model; (*c*) in the present paper taking into consideration the higher orders of approximation.

model of the critical behaviour of crystals with LR correlations caused by point defects with degenerate internal degrees of freedom [3, 4].

The models with LR-correlated quenched defects present both theoretical interest from the possibility of predicting new types of critical behaviour in disordered systems and experimental interest from the possibility of realizing RL-correlated defects in orientational glasses [5] and disordered solids containing fractal-like defects [3]. However, numerous investigations of pure and disordered systems performed with the use of the field-theoretic approach show that the predictions made in the one-loop approximation, especially on the basis of the ε-expansion, can differ strongly from the real critical behaviour [6–9]. Therefore, the map of regions with the various types of critical behaviour received for the WH model on the basis of ε, δ-expansion [1] (figure 1(*a*)) may not correspond to the critical behaviour of the 3D WH model for different values of m and a. In this case the results for the models with LR correlated defects received with the use of ε, δ-expansion [1, 3, 4, 10–12] must be corrected. To shed light on this question and to determine more accurately the dependence of the critical behaviour on the number of components of the order parameter m and the values of correlation parameter a, we have constructed a field-theoretical description of the 3D WH model in the two-loop approximation for the values of a in the range $2 \leqslant a \leqslant 3$.

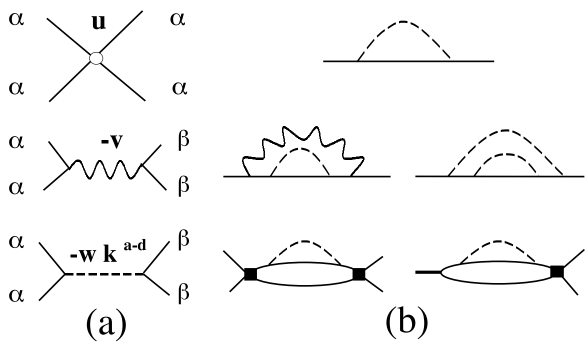

Figure 2. Graphs (*a*) that correspond to vertices u, v and w; (*b*) that, in addition, take into consideration the comparison with other works, using ε, δ-expansion, ■ corresponds to vertices u, v and w.

The effective Hamiltonian of the WH model after using the replica trick is given by

$$H_{\text{eff}} = \sum_{\alpha=1}^{n} \int \mathrm{d}^d x \left[\frac{1}{2}(r_0\phi_\alpha^2 + (\nabla\phi_\alpha)^2) + \frac{u_0}{4!}(\phi_\alpha^2)^2\right] - \sum_{\alpha,\beta=1}^{n} \int \mathrm{d}^d x\, \mathrm{d}^d y\, g(\boldsymbol{x}-\boldsymbol{y})\phi_\alpha^2(\boldsymbol{x})\phi_\beta^2(\boldsymbol{y}) \tag{1}$$

where $\phi_\alpha^2 = \sum_{i=1}^{m}\phi_{i\alpha}^2$; $\phi_{i\alpha}$ is an $(n \times m)$-component order parameter. The properties of the original disordered system are obtained in the replica number limit $n \to 0$. The Fourier transformation of the interaction vertex $g(x) \sim x^{-a}$ gives $g(k) = v_0 + w_0 k^{a-d}$ for small k. $g(k)$ must be positive definite, therefore if $a > d$, then the w term is irrelevant, $v_0 \geqslant 0$ and H_{eff} (1) corresponds to the model with SR-correlated defects, while if $a < d$, then the w term is dominant at small k and $w_0 \geqslant 0$.

As is known, in the field-theoretic approach [13] the asymptotic critical behaviour of systems in the fluctuation region are determined by the Callan–Symanzik renormalization-group equation for the vertex parts of the irreducible Green functions. To calculate the β functions and the critical exponents as functions of the renormalized interaction vertices u, v and w (scaling γ functions) appearing in the renormalization-group equation, we used the standard method based on the Feynmann diagram technique and the renormalization procedure [14]. The three types of interactions can be represented graphically as in figure 2(*a*). When we considered a diagrammatic representation of the two-point vertex function $\Gamma^{(2)}$, three types of four-point vertex functions $\Gamma_i^{(4)}$ and a two-point function with the ϕ^2 insertion vertex function $\Gamma^{(1,2)}$ in the two-loop approximation the diagrams were integrated numerically in $d = 3$ and with the values of parameter a determining the momentum dependence of the w interaction in the range $2 \leqslant a \leqslant 3$ with changes through the step $\Delta a = 0.01$. Details will be presented in a separate publication [15]. Unlike the works using ε, δ-expansion we took into consideration the graphs of the form (figure 2(*b*)), contributions of which are increased when the a values are removed from $a = 3$.

As a result, we obtained the β and γ functions in the two-loop approximation in the form of the expansion series in renormalized vertices u, v and w. Because of space restrictions presenting the coefficients of these series for different values of a we will only give here the obtained β and γ functions for $a = 2$ (the case with $a = 2$ corresponds to a system of straight lines of impurities or straight dislocation lines of random orientation in a sample):

$$
\begin{aligned}
\beta_u(u,v,w) = {} & -u + u^2 - \frac{3}{2}uv - 1.901\,416uw - \frac{4(41m+190)}{27(m+8)^2}u^3 + \frac{2(25m+131)}{27(m+8)}u^2 v \\
& -\frac{185}{216}uv^2 + \frac{(1.230\,378m + 6.713\,002)}{m+8}u^2 w \\
& -0.312\,654uw^2 - 1.193\,479uvw \\
\beta_v(u,v,w) = {} & v + v^2 + \frac{3}{2}w^2 + 1.901\,416vw - \frac{2(m+2)}{(m+8)}uv + \frac{95}{216}v^3 + 0.488\,229w^3 \\
& -\frac{50(m+2)}{27(m+8)}uv^2 - 1.974\,883\frac{(m+2)}{(m+8)}uw^2 + \frac{92(m+2)}{27(m+8)^2}u^2 v + 0.806\,375vw^2 \\
& +0.839\,125v^2 w - 1.939\,086\frac{(m+2)}{(m+8)}uvw \\
\beta_w(u,v,w) = {} & 2w + 0.628\,176w^2 + \frac{1}{2}vw - \frac{2(m+2)}{(m+8)}uw - 0.1528w^3 + \frac{92(m+2)}{27(m+8)^2}u^2 w \\
& +\frac{23}{216}v^2 w + 0.090\,516\frac{(m+2)}{(m+8)}uw^2 - 0.022\,629vw^2 - \frac{23(m+2)}{27(m+8)}uvw \\
\gamma_\phi(u,v,w) = {} & 0.004\,222w + \frac{8(m+2)}{27(m+8)^2}u^2 + \frac{1}{108}v^2 + 0.056\,893w^2 - \frac{2(m+2)}{27(m+8)}uv \\
& -0.315\,823\frac{(m+2)}{(m+8)}uw + 0.078\,956vw \\
\gamma_{\phi^2}(u,v,w) = {} & -\frac{m+2}{m+8}u + \frac{1}{4}v + 0.318\,31w + \frac{2(m+2)}{(m+8)^2}u^2 + \frac{1}{16}v^2 - 0.019\,507w^2 \\
& -\frac{(m+2)}{2(m+8)}uv - 0.270\,565\frac{(m+2)}{(m+8)}uw + 0.067\,641vw.
\end{aligned}
\tag{2}
$$

The series (2) are normalized by a standard change of variables [7, 8] $u \to 6u/(m+8)J$, $v \to v/32J$, $w \to w/32J$, so that the coefficients of the terms u, u^2 and v, v^2 in β_u and β_v become 1 in modulus, where $J = \int \mathrm{d}^d q/(q^2+1)^2$ is the one-loop integral.

The nature of the critical behaviour is determined by the existence of a stable FP satisfying the system of equations

$$\beta_i(u^*, v^*, w^*) = 0 \qquad (i = 1, 2, 3). \tag{3}$$

It is well known that perturbation series are asymptotically convergent, and the vertices describing the interaction of the order parameter fluctuations in the fluctuating region $r \to 0$ are large enough so that expressions (2) cannot be used directly. For this reason, to extract the required physical information from the obtained expressions, we employed the Padé–Borel approximation of the summation of asymptotically convergent series extended to the multiparameter case [9, 16]. We used the [2/1] approximant to calculate the β functions in the two-loop approximation.

However, the analysis of the series coefficients for the β_w function has shown that the summation of this series is fairly poor, which resulted in the absence of FP with $w^* \neq 0$, for example, in the case $m = 1$ for $a < 2.93$, in the case $m = 2$ for $a < 2.67$ etc. Dorogovtsev found the symmetry of the scaling function for the WH model in relation to the transformation $(u, v, w) \to (u, v, v + w)$ [10] which gives the possibility of investigating the problem of FP existence with $w^* \neq 0$ in the variables $(u, v, v + w)$. In this case our investigations have shown the existence of FPs with $w^* \neq 0$ in the whole region where the parameter a changes.

We have found three types of FPs in the physical region of parameter space $u^*, v^*, v^* + w^* \geqslant 0$ for different values of m and a. Type I corresponds to the FP of a pure system ($u^* \neq 0, v^*, w^* = 0$), type II is a SR-disorder FP ($u^*, v^* \neq 0, w^* = 0$) and type III corresponds to LR-disorder FPs ($u^*, v^*, w^* \neq 0$). The type of critical behaviour of this

Table 1. Stable fixed points of the 3D WH model from two-loop expansions.

	$m = 1$			$m = 2$			$m = 3$		
a	u^*	v^*	$w^* + v^*$	u^*	v^*	$w^* + v^*$	u^*	v^*	$w^* + v^*$
3.1	2.383 38	0.551 64	0.551 64	1.564 69	0.004 16	0.004 16	1.520 97	0.000 00	0.000 00
3.0	2.383 38	0.222 93	0.551 64	1.564 69	0.004 16	0.004 16	1.520 97	0.000 00	0.000 00
2.9	2.598 04	0.318 90	0.681 14	2.090 01	0.113 86	0.400 38	1.520 97	0.000 00	0.000 00
2.8	2.779 27	0.401 53	0.782 99	2.176 77	0.135 36	0.443 59	1.957 70	0.082 98	0.345 50
2.7	2.940 31	0.474 87	0.867 57	2.267 78	0.159 23	0.486 12	2.017 46	0.093 46	0.370 04
2.6	3.086 45	0.540 84	0.939 16	2.360 58	0.184 57	0.526 33	2.086 99	0.109 22	0.400 05
2.5	3.219 83	0.600 35	0.999 72	2.496 43	0.234 42	0.596 51	2.155 85	0.125 35	0.426 28
2.4	3.340 78	0.653 74	1.049 98	2.618 18	0.280 94	0.653 34	2.220 47	0.140 74	0.446 51
2.3	3.448 13	0.700 82	1.089 80	2.725 20	0.323 44	0.697 60	2.308 01	0.169 10	0.483 02
2.2	3.538 99	0.740 92	1.118 25	2.815 01	0.361 15	0.729 09	2.392 98	0.200 79	0.516 96
2.1	3.608 14	0.772 63	1.133 40	2.883 05	0.392 93	0.746 72	2.458 69	0.228 77	0.537 59
2.0	3.646 87	0.793 47	1.131 89	2.922 06	0.417 10	0.748 43	2.499 45	0.251 61	0.543 64

disordered system for each value of m and a is determined by the stability of the corresponding FP. The requirement that the FP be stable reduces to the condition that the eigenvalues of the matrix

$$B_{i,j} = \frac{\partial \beta_i(u_1^*, u_2^*, u_3^*)}{\partial u_j} \tag{4}$$

lie in the right-hand side complex half-plane.

Values of the stable FPs obtained for the most interesting values of the number of order-parameter components m and $2 \leqslant a \leqslant 3$ are presented in table 1. As one can see from this table, for the Ising model ($m = 1$) the LR-disorder FP is stable for values of a in the whole investigated range. The additional calculations for $3 < a < 4$ have shown that only FP II is stable in this range. For $a = 3$ FP values for vertices u and $g(k)$ are equal, $u^* = 2.383\,38$ and $g^* = v^* + w^* = 0.551\,64$, and correspond to the SR-disordered Ising model FP, although $w^* \neq 0$. Similarly, for $m = 1$ and $a = 3$ the LR disorder is marginal, and the critical behaviour of the WH model, as that of the SR-disordered Ising model, is characterized by the same critical exponents (table 2). The critical behaviour of the XY-model ($m = 2$) is determined by the LR-disorder FP for $a \leqslant 2.96$ and the SR-disorder FP for $a > 2.96$. The Heisenberg model ($m = 3$) is characterized by a change in the types of critical behaviour from the LR-disorder type (III) for $a \leqslant 2.85$ to the pure type (I) for $a > 2.85$. Figure 1(b) shows regions of the various types of critical behaviour of the WH model, which we obtained in the two-loop approximation. The large change in the picture indicates that the correspondence between the WH results and our calculations in the two-loop approximation is weak.

However, the results which we received for the disordered XY-model must be corrected. We believe that in the higher field-theory orders of approximation k the critical behaviour of the XY-model will be determined by the FP of pure type (I) for $a_c^{(k)} < a$, but not by the SR-disorder FP (II), obtained in the two-loop order. Here, $a_c^{(k)}$ is a marginal value for a in the kth order of approximation, for which disorder is irrelevant ($a_c^{(6)} \simeq 2/\nu_o = 2.99$ with $\nu_o = 0.669$ [17] for $m = 2$). Two facts indicate this, such as the weak stability of the SR-disorder FP revealed for $2.96 < a < 4$ and that $a_c^{(2)} = 3$ for $m_c = 2.0114$. In the higher orders of approximation the marginal value of m_c can be found with the use of the Harris criterion [2] $d\nu_o(m_c) - 2 = 0$, and as $\nu_o = 0.669$ [17] for $m = 2$, then $m_c < 2$. Therefore, we believe that the corrected picture of the regions of various types of critical behaviour of the model with LR-correlated defects will be represented by figure 1(c).

Table 2. Critical exponents of the 3D WH model from two-loop expansions.

		$m = 1$			$m = 2$			$m = 3$		
a	$2/a$	η	ν	z	η	ν	z	η	ν	z
3.1		0.0327	0.6715	2.1712	0.0288	0.6642	2.0000	0.0283	0.6960	2.0217
3.0	0.6667	0.0327	0.6715	2.1712	0.0288	0.6642	2.0000	0.0283	0.6960	2.0217
2.9	0.6897	0.0304	0.6813	2.2120	0.0248	0.7141	2.1315	0.0283	0.6960	2.0217
2.8	0.7143	0.0270	0.6889	2.2486	0.0212	0.7190	2.1510	0.0179	0.7600	2.1128
2.7	0.7407	0.0227	0.6950	2.2837	0.0166	0.7240	2.1736	0.0137	0.7632	2.1269
2.6	0.7692	0.0176	0.7002	2.3184	0.0112	0.7692	2.1988	0.0084	0.7682	2.1443
2.5	0.8000	0.0118	0.7046	2.3532	0.0035	0.7378	2.2338	0.0025	0.7727	2.1633
2.4	0.8333	0.0055	0.7083	2.3879	−0.0050	0.7452	2.2684	−0.0040	0.7763	2.1827
2.3	0.8696	−0.0012	0.7114	2.4215	−0.0138	0.7513	2.3013	−0.0125	0.7835	2.2078
2.2	0.9091	−0.0081	0.7137	2.4524	−0.0226	0.7558	2.3301	−0.0218	0.7905	2.2315
2.1	0.9524	−0.0147	0.7151	2.4780	−0.0307	0.7588	2.3522	−0.0303	0.7952	2.2514
2.0	1.0000	−0.0205	0.7155	2.4949	−0.0371	0.7599	2.3649	−0.0370	0.7975	2.2644

Finally, we have calculated the static critical exponents for the WH model (table 2), received from the resummed by the generalized Padé–Borel method γ functions in the corresponding stable FPs: $\eta = \gamma_\phi(u^*, v^*, w^*)$, $\nu = [2 + \gamma_{\phi^2}(u^*, v^*, w^*) - \gamma_\phi(u^*, v^*, w^*)]^{-1}$. Also, we have found by the method used in [8] the dynamic scaling function γ_λ and calculated the values of the dynamic exponent $z = 2 + \gamma_\lambda(u^*, v^*, w^*)$ on the basis of the resummed γ_λ function (table 2). As an example, for $a = 2$ the received γ_λ function is given by

$$\gamma_\lambda(u, v, w) = \frac{1}{4}v + 0.314\,088w + 0.226\,777\frac{(m+2)}{(m+8)^2}u^2 + \frac{23}{432}v^2 - 0.0764w^2 \\ -0.092\,593\frac{(m+2)}{(m+8)}uv + 0.123\,604\frac{(m+2)}{(m+8)}uw - 0.011\,315vw. \quad (5)$$

The comparison of the exponent ν values and ratio $2/a$ from table 2 shows the violation of supposed in [1] on the basis of some heuristic arguments as exact the relation $\nu = 2/a$. The revealed difference is caused by the use in our work of a more accurate field-theoretic description in the higher orders of approximation for the 3D system directly together with methods of series summation. Also, these distinctions can be explained by the application for calculations of the concrete numerical values of parameter a and taking into consideration the graphs of the form shown in figure 2(*b*), thrown away when the ε, δ-expansion is used, but contributions of which are increased when the values a are removed from $a = 3$. Of course, there are errors in the present consideration determined by the accuracy of series summation for the β and γ functions. However, comparison of the exponent values for the SR-disorder Ising model, calculated with the use of the Padé–Borel method in [6, 7] in the two-loop and four-loop approximations respectively, shows that their differences are not more than 0.02. At the same time, in our work $\nu - 2/a$ depends on the values of a and m and has the value 0.284, as example, for $a = 2$ and $m = 1$, which is considerably larger.

In closing, we hope that the features of the critical behaviour of the WH model revealed in our paper will stimulate the organization of experimental works in real disordered systems with long-range correlated defects like orientational glasses and solids with fractal-like defects. Also, computational methods can be applied to simulate disordered systems with straight lines of impurities of random orientation in a sample ($a = 2$). The received values of exponents can be used for an explanation of the results of a computer simulation of the 3D disordered Ising model [18] at impurity concentrations between the threshold of impurity percolation and the spin-percolation threshold, in which the fractal-like behaviour of impurity-extended structures and the competition between impurity-percolating and spin-percolating clusters are possible.

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